Exercise 36

Find $f'(a)$.

$$f(x)=\frac{4}{\sqrt{1-x}}$$

Solution

Determine the derivative of $f(x)$.

$$\begin{aligned}
f'(x) &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} \\
&= \lim_{h\to 0}\frac{\frac{4}{\sqrt{1-(x+h)}}-\frac{4}{\sqrt{1-x}}}{h} \\
&= \lim_{h\to 0}\frac{\frac{4}{\sqrt{1-x-h}}-\frac{4}{\sqrt{1-x}}}{h} \\
&= \lim_{h\to 0}\frac{\frac{4\sqrt{1-x}}{\sqrt{1-x}\sqrt{1-x-h}}-\frac{4\sqrt{1-x-h}}{\sqrt{1-x}\sqrt{1-x-h}}}{h} \\
&= \lim_{h\to 0}\frac{\frac{4\sqrt{1-x}-4\sqrt{1-x-h}}{\sqrt{1-x}\sqrt{1-x-h}}}{h} \\
&= \lim_{h\to 0}\frac{4\sqrt{1-x}-4\sqrt{1-x-h}}{h\sqrt{1-x}\sqrt{1-x-h}} \\
&= \lim_{h\to 0}\frac{4\sqrt{1-x}-4\sqrt{1-x-h}}{h\sqrt{1-x}\sqrt{1-x-h}}\cdot\frac{4\sqrt{1-x}+4\sqrt{1-x-h}}{4\sqrt{1-x}+4\sqrt{1-x-h}} \\
&= \lim_{h\to 0}\frac{\left(4\sqrt{1-x}-4\sqrt{1-x-h}\right)\left(4\sqrt{1-x}+4\sqrt{1-x-h}\right)}{h\sqrt{1-x}\sqrt{1-x-h}\left(4\sqrt{1-x}+4\sqrt{1-x-h}\right)} \\
&= \lim_{h\to 0}\frac{[16(1-x)-16(1-x-h)]}{h\sqrt{1-x}\sqrt{1-x-h}\left(4\sqrt{1-x}+4\sqrt{1-x-h}\right)} \\
&= \lim_{h\to 0}\frac{16h}{h\sqrt{1-x}\sqrt{1-x-h}\left(4\sqrt{1-x}+4\sqrt{1-x-h}\right)} \\
&= \lim_{h\to 0}\frac{16}{\sqrt{1-x}\sqrt{1-x-h}\left(4\sqrt{1-x}+4\sqrt{1-x-h}\right)} \\
&= \frac{16}{\sqrt{1-x}\sqrt{1-x}\left(4\sqrt{1-x}+4\sqrt{1-x}\right)} \\
&= \frac{16}{(1-x)\left(8\sqrt{1-x}\right)} \\
&= \frac{2}{(1-x)^{3/2}}
\end{aligned}$$

Plug in $x = a$ to this formula to get $f'(a)$.

$$f'(a) = \frac{2}{(1-a)^{3/2}}$$